Elliptic equations with surface measures

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Abstract

In 1812, S. D. Poisson has examined how electric charges generate an electric field $\vec{E}$. This is how the famous *Poisson equation* $-\Delta u = f$ came into mathematics. The electric field is computed via $\vec{E} = \nabla u$.

Poisson assumed that the electric charges lie on a $3D$-object (a *charge carrier*) and the field lines spread into a $3D$-domain. If the charge carrier is however very thin, one might as well want to look at it as a 2-dimensional surface $\Gamma \subset \mathbb{R}^3$.

While $3D$-charge carriers can be modeled by a function f, the $2D$-setting gives rise to *(Hausdorff) measures*, leading to the equation $-\Delta u = \mathcal{H}^2 \llcorner \Gamma$, which we will explain.

The questions we examine is whether in this setting the electric field $\vec{E} = \nabla u$ is *bounded/continuous* close to the charge carrier Γ. The answers depend highly on the geometry of Γ.

Mathematically, this question is thought of as a question of *regularity* for a *PDE involving measures.*